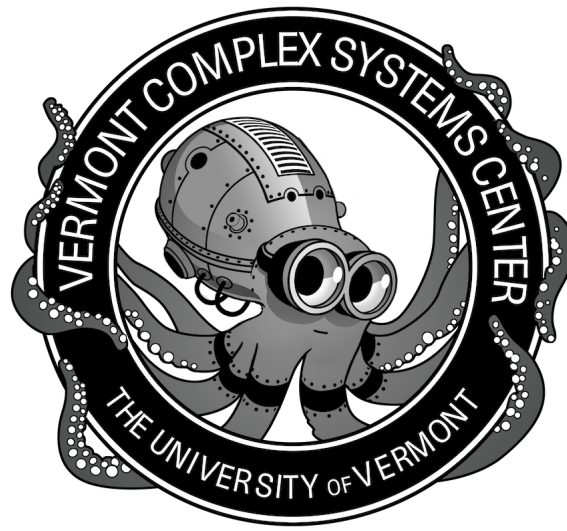


# Dominance Hierarchy in the Network of Hockey Fighters

Randall Harp, Laurent Hébert-Dufresne,  
Daniel B. Larremore, and Antonio D. Sirianni



## What are hockey fights?

When two hockey teams play each other, referees enforce the rules in the rulebook by issuing penalties, but the players themselves enforce the cultural norms of the game through fights. For instance, when a player on one team disrespects the opposing team's goalie, shoots the puck into the goal after the play has stopped, or dangerously slams another player into the boards, a fight is likely to occur. In the National Hockey League (NHL; U.S. & Canada), nearly half (46%) of games have one or more fights!

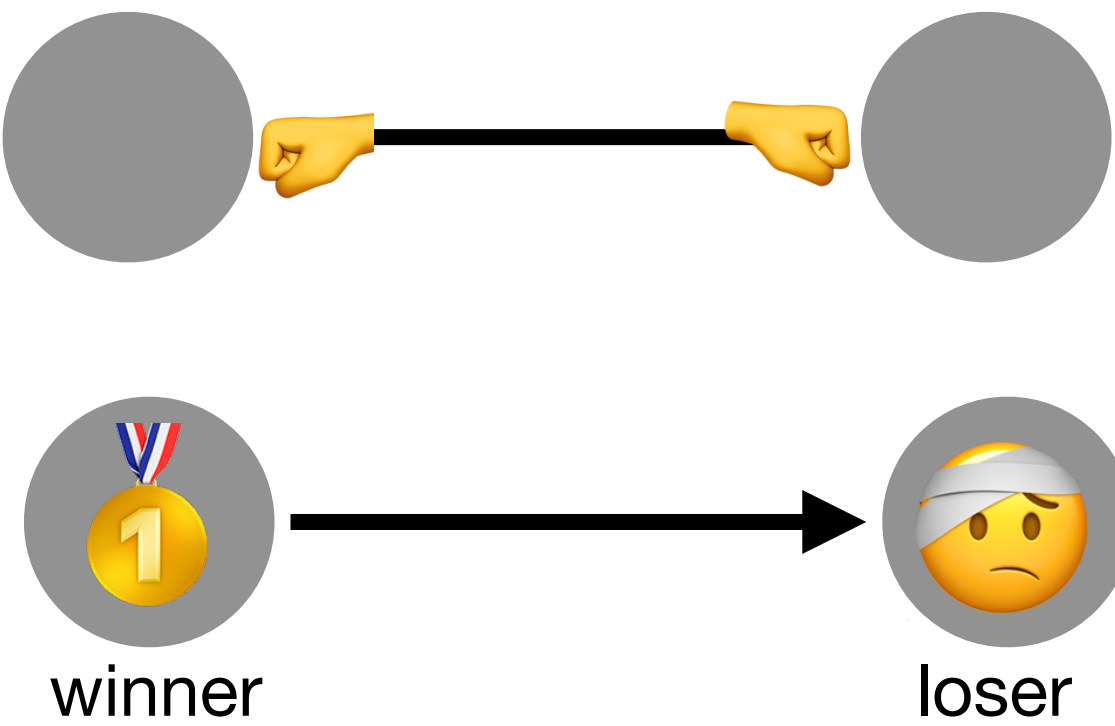


## Big hits, big questions.

Who fights whom? Who wins? What are the consequences for the players and for the game? And are enforcers actually better at fighting or do they simply fight more often? In short, what is the structure of the network of hockey fights between players, and what does it tell us about the impact of fighting on the trajectories of games?

## A Network of Hockey Fights.

**Data:** We analyzed over 10 seasons of fights from [HockeyFights.com](http://HockeyFights.com). Users of this site watch televised hockey fights and vote on who won. From this, we build a network.



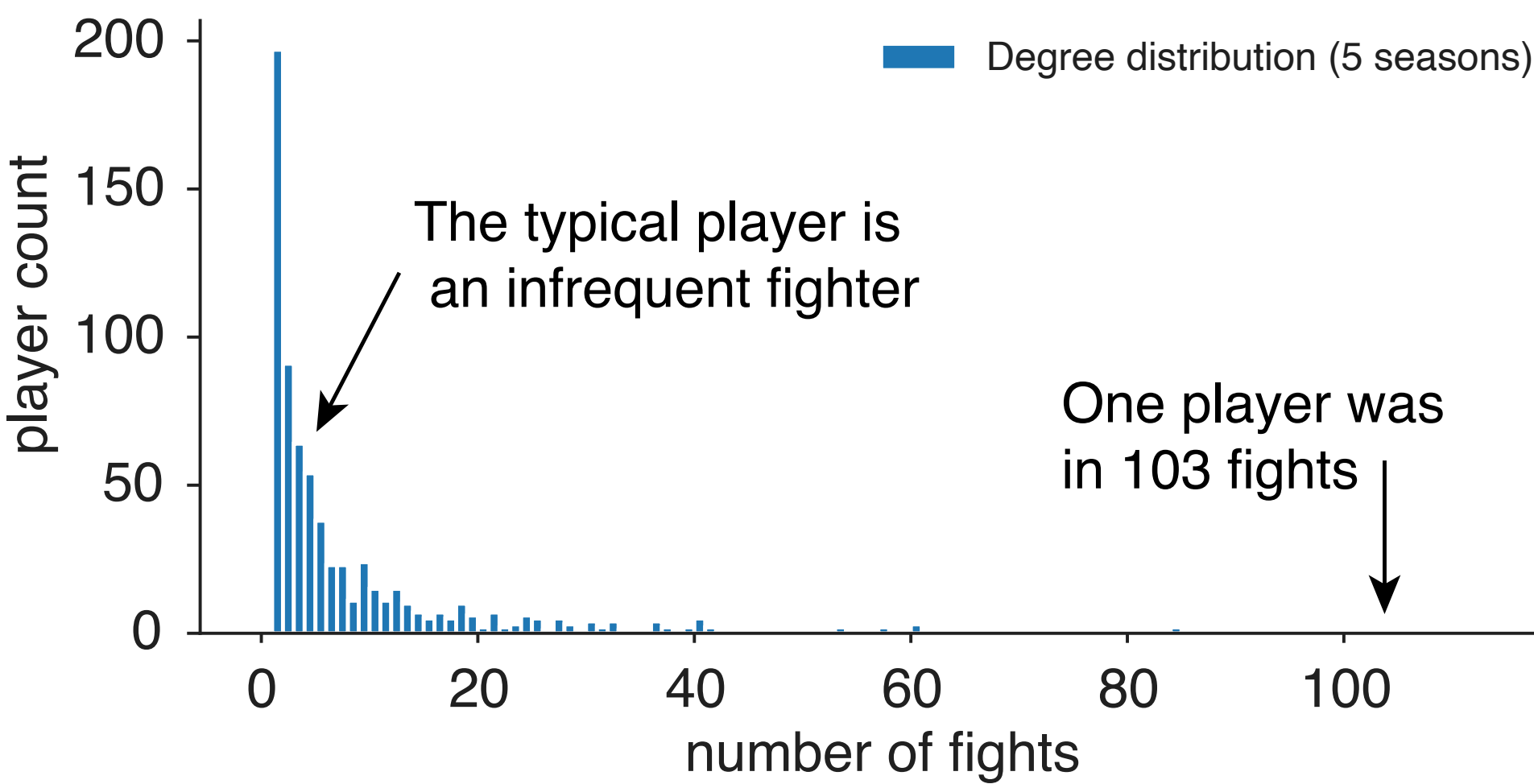
Wins and losses form a **directed multigraph of hockey fights**.

For each fight, we identified the players, the time of the fight, and used the votes of hockey fans to label which player won or if the fight was a draw. We constructed a network from that dataset in which each node represents a player and each directed, weighted edge represents the directional outcome of the fight; in a draw, edges of weight 1/2 were placed in both directions between players. Thus, the hockey fights network is a time-stamped, weighted, directed multigraph.

## Edge existence: who fights, and how often?

### Edge generating mechanisms.

Some players have the specialized role of *enforcer*, and are expected to fight on behalf of their team, leading to lopsided matchups between players who fight often and those who do not. At times, however, an offending player is sufficiently valuable that their injury in a fight cannot be risked, and thus the offending team's enforcer will step in to fight another enforcer. Due to positive response from hockey fans, players may even spontaneously fight.



### Heavy hits, heavy tails.

Over 5 seasons of data, most players fight only a handful of times. And yet, one player was in 103 fights. That means that guy fought once every 4 games, on average!

This analysis applies only to the existence and formation of edges, not to the direction.

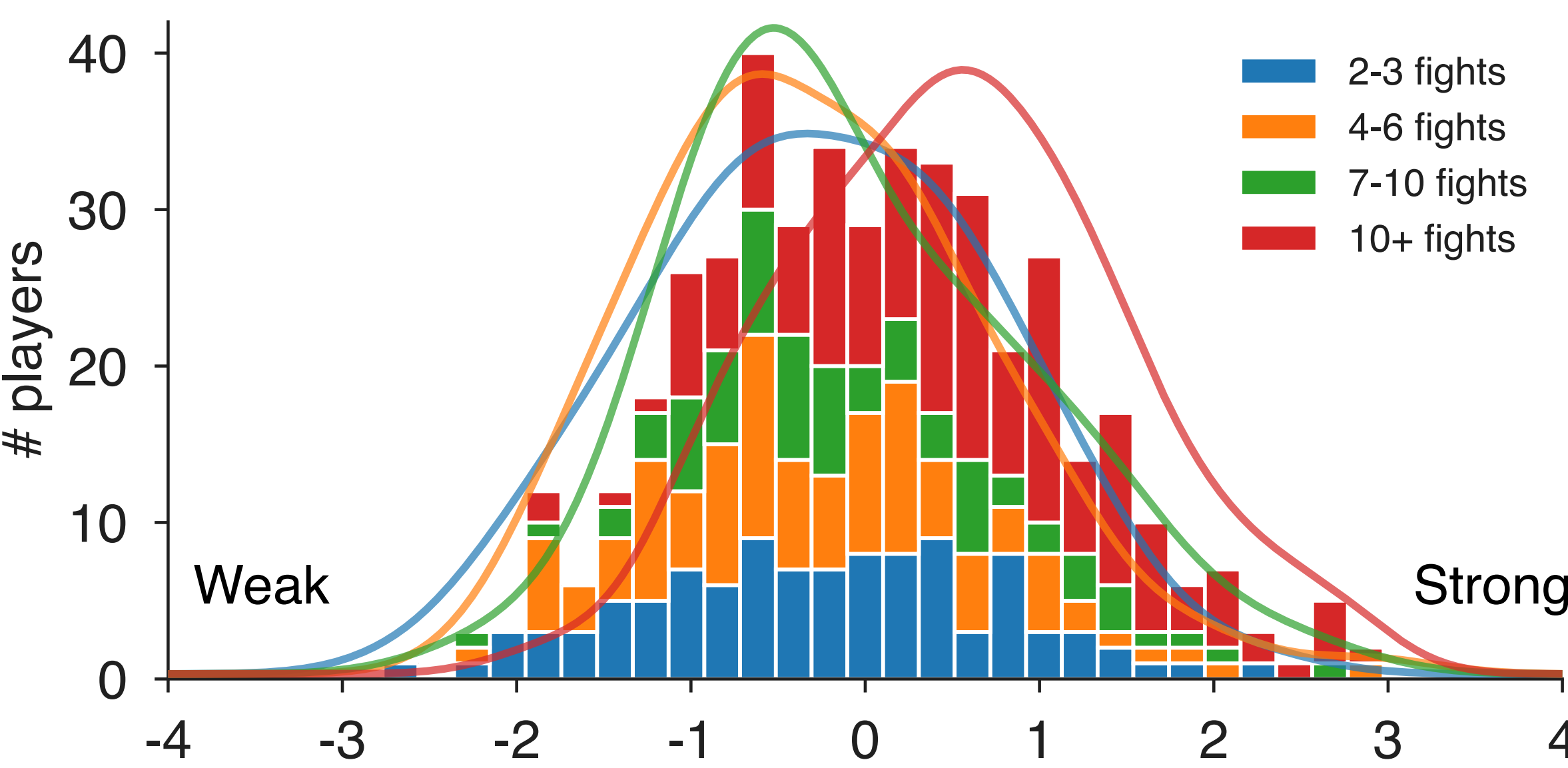
## Edge direction and ranking: who wins?

### Edge direction formation mechanisms.

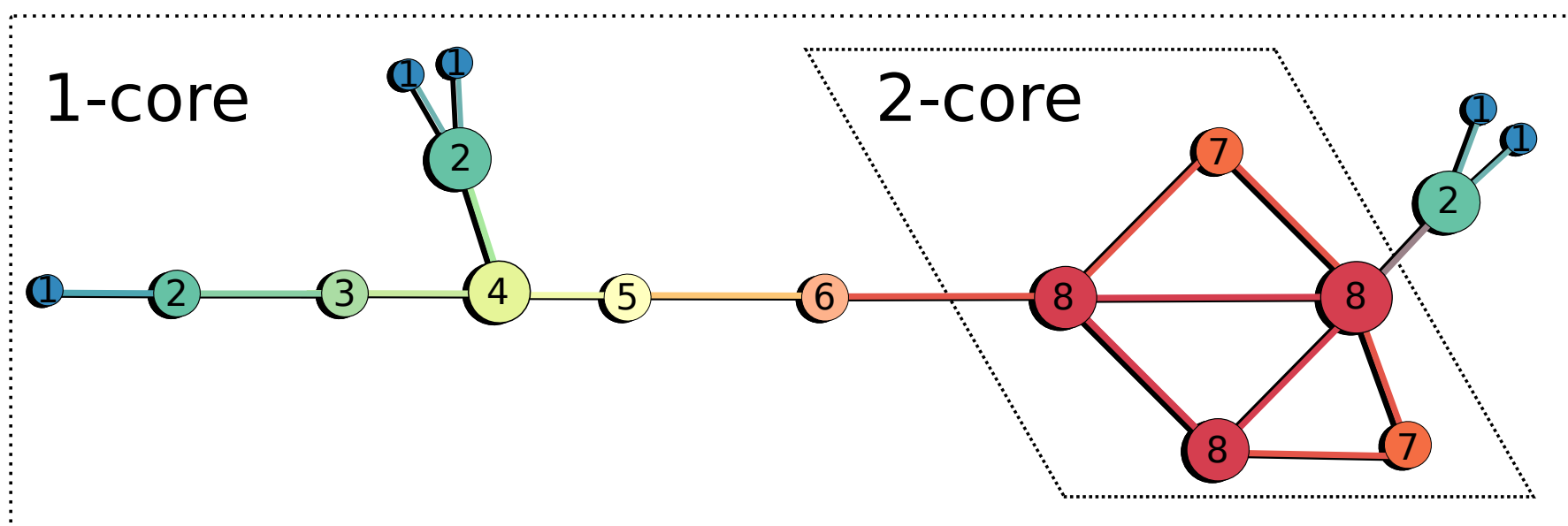
The directed, weighted, multigraph of hockey fights can be used to estimate player strengths via SpringRank. An  $n$ -unit difference denotes  $3^n$ -to-1 odds that the higher ranked player wins. Kernel density estimates show that players who fight more often are indeed more skilled.

### Specialization: fight often, fight better.

Hockey players who fight more often are also stronger fighters. While the causal arrows may point in either direction (better fighters fight more -vs- frequent fighters gain skills), this pattern indicates specialization: players who fight often (10+, red) are ranked one rank higher than others on average—a 75% win rate.

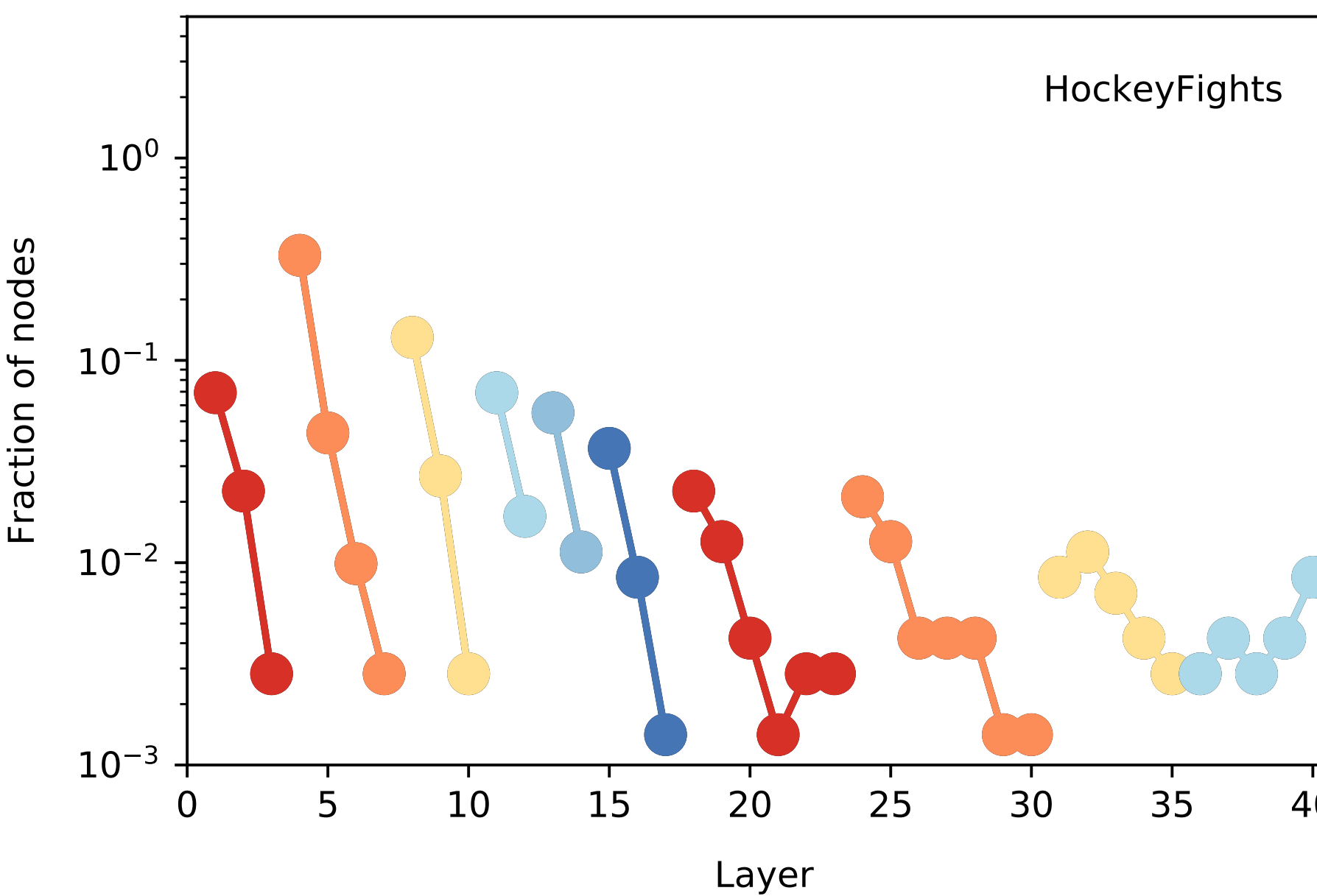


## Centrality: peeling the onion

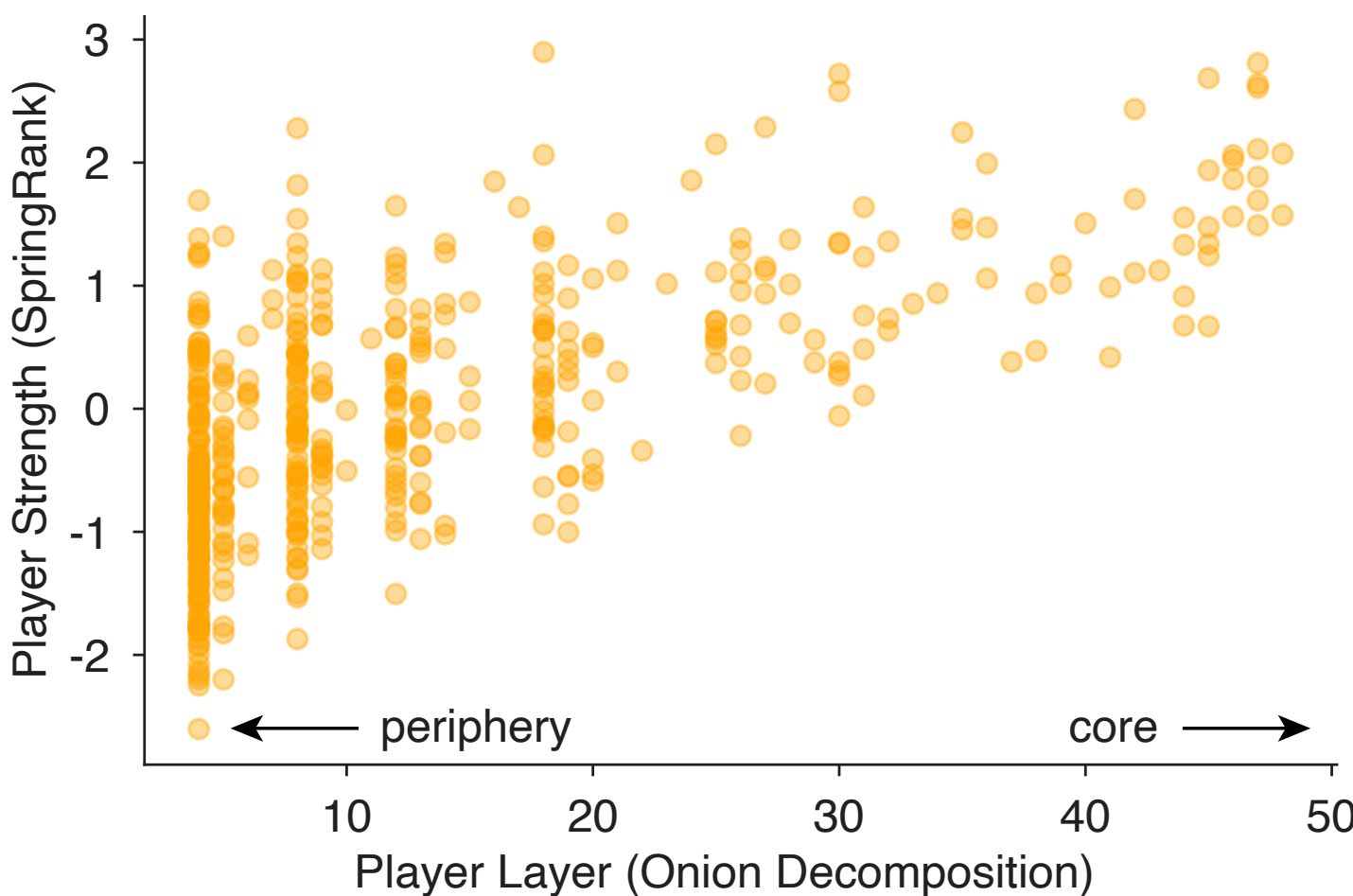


### Who are the central fighters in the Hockey Fights Network?

The quality of a hockey fighter is a multidimensional quantity; depending on both raw talent and experience. We introduce a directed onion decomposition, a refinement of the  $k$ -core decomposition, where we iteratively peel nodes of out-degrees (wins) less than some core value  $k$ . The layer of a node then corresponds to the step (or onion peel) in which they were removed. This centrality metric balances wins (out-degree) with quality of the opponent (layer of the loser).



## Core players are more powerful



**Centrality versus Strengths.** We compare the network layer in which we find a player with their strength evaluated from the SpringRank algorithm. The former metric only aims to evaluate the centrality of a player while the latter attempts to predict the outcome of observed fights through an inferred ranking. Despite this difference, we find strong correlations between both metrics. Suggesting fighters work their way up the ladder, fighting opponents matching their own power.

## Contact, Code, Paper

### preprint

Forthcoming! Stay tuned.

### contact

Daniel.Larremore@colorado.edu  
Laurent.Hebert-Dufresne@uvm.edu

